NAG Toolbox for MATLAB

g13fg

1 Purpose

g13fg estimates the parameters of a univariate regression-exponential GARCH(p, q) process (see Engle and Ng 1993).

2 Syntax

```
[theta, se, sc, covr, hp, et, ht, lgf, ifail] = g13fg(dist, yt, x, ip,
iq, nreg, mn, theta, hp, copts, maxit, tol, 'num', num, 'npar', npar)
```

3 Description

A univariate regression-exponential GARCH(p, q) process, with q coefficients α_i, for $i = 1, \ldots, q$, q coefficients ϕ_i, for $i = 1, \ldots, q$, p coefficients, β_i, for $i = 1, \ldots, p$ and k linear regression coefficients b_i, for $i = 1, \ldots, k$, can be represented by:

$$ln(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i z_{t-i} + \sum_{j=1}^{q} \phi_i \left(|z_{t-j}| - E[|z_{t-i}|] \right) + \sum_{j=1}^{p} \beta_i ln\left(h_{t-j}\right), \qquad t = 1, \ldots, T \qquad (1)$$

where $z_t = \frac{\epsilon_t}{\sqrt{h_t}}$, $E[|z_{t-i}|]$ denotes the expected value of $|z_{t-i}|$ and $\epsilon_t \mid \psi_{t-1} = N(0, h_t)$ or $\epsilon_t \mid \psi_{t-1} = S_t(df, h_t)$. Here S_t is a standardized Student's t-distribution with df degrees of freedom and variance h_t, T is the number of terms in the sequence, y_t denotes the endogenous variables, x_t the exogenous variables, b_o the regression mean, b the regression coefficients, ϵ_t the residuals, h_t is the conditional variance, df the number of degrees of freedom of the Student's t-distribution, and ψ_t the set of all information up to time t.

g13fg provides an estimate for $\hat{\theta}$, the vector $\theta = \left(b_o, b^{\mathrm{T}}, \omega^{\mathrm{T}}\right)$ where $b^{\mathrm{T}} = (b_1, \ldots, b_k)$, $\omega^{\mathrm{T}} = \left(\alpha_0, , , \alpha_1, , , \ldots, , , \alpha_q, , , \phi_1, , , \ldots, , , \phi_q, , , \phi_1, , , \ldots, , , \phi_q, \gamma\right)$ when **dist** = 'N', and $\omega^{\mathrm{T}} = \left(\alpha_0, , , \alpha_1, , , \ldots, , , \alpha_q, , , \phi_1, , , \ldots, , , \phi_q, , , \phi_1, , , \ldots, , , \phi_q, , , \gamma, df\right)$ when **dist** = 'T'.

mn, **nreg** can be used to simplify the GARCH(p, q) expression in (1) as follows:

No Regression and No Mean

$y_t = \epsilon_t$,

mn $= 0$,

nreg $= 0$ and

θ is a $(2 \times q + p + 2)$ vector when **dist** = 'N', and a $(2 \times q + p + 3)$ vector, when **dist** = 'T'.

No Regression

$y_t = b_o + \epsilon_t$,

mn $= 1$,

nreg $= 0$ and

θ is a $(2 \times q + p + 3)$ vector when **dist** = 'N' and a $(2 \times q + p + 4)$ vector, when **dist** = 'T'.

Note: if the $y_t = \mu + \epsilon_t$, where μ is known (not to be estimated by g13fg) then (1) can be written as $y_t^{\mu} = \epsilon_t$, where $y_t^{\mu} = y_t - \mu$. This corresponds to the case **No Regression and No Mean**, with y_t replaced by $y_t - \mu$.

No Mean

$y_t = x_t^{\mathrm{T}} b + \epsilon_t$,

$\mathbf{mn} = 0$,

$\mathbf{nreg} = k$ and

θ is a $(2 \times q + p + 2 + k)$ vector when **dist** = 'N' and a $(2 \times q + p + 3 + k)$ vector, when **dist** = 'T'.

4 References

Bollerslev T 1986 Generalised autoregressive conditional heteroskedasticity *Journal of Econometrics* **31** 307–327

Engle R 1982 Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation *Econometrica* **50** 987–1008

Engle R and Ng V 1993 Measuring and Testing the Impact of News on Volatility *Journal of Finance* **48** 1749–1777

Glosten L, Jagannathan R and Runkle D 1993 Relationship between the expected value and the volatility of nominal excess return on stocks *Journal of Finance* **48** 1779–1801

Hamilton J 1994 *Time Series Analysis* Princeton University Press

5 Parameters

5.1 Compulsory Input Parameters

1: **dist – string**

The type of distribution to use for e_t.

dist = 'N'

A Normal distribution is used.

dist = 'T'

A Student's t-distribution is used.

Constraint: **dist** = 'N' or 'T'.

2: **yt(num) – double array**

The sequence of observations, y_t, for $t = 1, \ldots, T$.

3: **x(ldx,*) – double array**

The first dimension of the array **x** must be at least **num**

The second dimension of the array must be at least $\max(1, \mathbf{nreg} + \mathbf{mn})$

Row t of **x** must contain the time dependent exogenous vector x_t, where $x_t^{\mathrm{T}} = \left(x_t{}^1, \ldots, x_t{}^k\right)$, for $t = 1, \ldots, T$.

4: **ip – int32 scalar**

The number of coefficients, β_i, for $i = 1, \ldots, p$.

Constraint: $\mathbf{ip} \geq 0$(see also **npar**)

5: **iq – int32 scalar**

The number of coefficients, α_i, for $i = 1, \ldots, q$.

Constraint: $\mathbf{iq} \geq 1$(see also **npar**)

6: **nreg – int32 scalar**

k, the number of regression coefficients.

Constraint: **nreg** ≥ 0(see also **npar**)

7: **mn – int32 scalar**

If **mn** $= 1$, the mean term b_0 will be included in the model.

Constraint: **mn** $= 0$ or 1.

8: **theta**(**npar**) **– double array**

The initial parameter estimates for the vector θ.

The first element must contain the coefficient α_o and the next **iq** elements must contain the autoregressive coefficients α_i, for $i = 1, \ldots, q$.

The next **iq** elements contain the coefficients ϕ_i, for $i = 1, \ldots, q$.

The next **ip** elements must contain the moving average coefficients β_j, for $j = 1, \ldots, p$.

If **dist** = 'T', the next element must contain an estimate for df, the number of degrees of freedom of the Student's t-distribution.

If **mn** $= 1$, the next element must contain the mean term b_o.

If **copts** = **false**, the remaining **nreg** elements are taken as initial estimates of the linear regression coefficients b_i, for $i = 1, \ldots, k$.

9: **hp – double scalar**

If **copts** = **false** then **hp** is the value to be used for the pre-observed conditional variance, otherwise**hp** is not referenced.

10: **copts – logical scalar**

If **copts** = **true** then the function provides initial parameter estimates of the regression terms, otherwise these are provided by you.

11: **maxit – int32 scalar**

The maximum number of iterations to be used by the optimization function when estimating the GARCH(p, q) parameters. If **maxit** is set to 0 then the standard errors, score vector and variance-covariance are calculated for the input value of θ in **theta**; however the value of θ is not updated.

Constraint: **maxit** > 0.

12: **tol – double scalar**

The tolerance to be used by the optimization function when estimating the GARCH(p, q) parameters.

5.2 Optional Input Parameters

1: **num – int32 scalar**

Default: The dimension of the arrays **yt**, **et**, **ht**. (An error is raised if these dimensions are not equal.)

T, the number of terms in the sequence.

Constraint: **num** $\geq \max(\mathbf{ip}, \mathbf{iq})$.

2: **npar – int32 scalar**

Default: The dimension of the arrays **theta**, **se**, **sc**, **covar**. (An error is raised if these dimensions are not equal.)

the number of parameters to be included in the model. $\mathbf{npar} = 1 + 2 \times \mathbf{iq} + \mathbf{ip} + \mathbf{mn} + \mathbf{nreg}$ when $\mathbf{dist} =$ 'N' and $\mathbf{npar} = 2 + 2 \times \mathbf{iq} + \mathbf{ip} + \mathbf{mn} + \mathbf{nreg}$ when $\mathbf{dist} =$ 'T'.

Constraint: $\mathbf{npar} < 20$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldx, ldcovr, work, lwork

5.4 Output Parameters

1: **theta(npar) – double array**

The estimated values $\hat{\theta}$ for the vector θ.

The first element contains the coefficient α_o, next **iq** elements contain the coefficients ϕ_i, for $i = 1, \ldots, q$.

The next **ip** elements are the moving average coefficients β_j, for $j = 1, \ldots, p$.

If $\mathbf{dist} =$ 'T', the next element contains an estimate for df then the number of degrees of freedom of the Student's t-distribution.

If $\mathbf{mn} = 1$, the next element contains an estimate for the mean term b_o.

The final **nreg** elements are the estimated linear regression coefficients b_i, for $i = 1, \ldots, k$.

2: **se(npar) – double array**

The standard errors for $\hat{\theta}$.

The first element contains the standard error for α_o, the next **iq** elements contain the standard errors for α_i, for $i = 1, \ldots, q$, the next **iq** elements contain the standard errors for ϕ_i, for $i = 1, \ldots, q$, the next **ip** elements are the standard errors for β_j, for $j = 1, \ldots, p$.

If $\mathbf{dist} =$ 'T', the next element contains the standard error for df, the number of degrees of freedom of the Student's t-distribution.

If $\mathbf{mn} = 1$, the next element contains the standard error for b_o.

The final **nreg** elements are the standard errors for b_j, for $j = 1, \ldots, k$.

3: **sc(npar) – double array**

The scores for $\hat{\theta}$.

The first element contains the scores for α_o, the next **iq** elements contain the scores for α_i, for $i = 1, \ldots, q$, the next **iq** elements contain the scores for ϕ_i, for $i = 1, \ldots, q$, the next **ip** elements are the scores for β_j, for $j = 1, \ldots, p$.

If $\mathbf{dist} =$ 'T', the next element contains the scores for df, the number of degrees of freedom of the Student's t-distribution.

If $\mathbf{mn} = 1$, the next element contains the score for b_o.

The final **nreg** elements are the scores for b_j, for $j = 1, \ldots, k$.

4: **covr(ldcovr,npar) – double array**

The covariance matrix of the parameter estimates $\hat{\theta}$, that is the inverse of the Fisher Information Matrix.

5: **hp – double scalar**

If **copts** $=$ **true** then **hp** is the estimated value of the pre-observed conditional variance.

6: **et**(**num**) **– double array**

The estimated residuals, ϵ_t, for $t = 1, \ldots, T$.

7: **ht**(**num**) **– double array**

The estimated conditional variances, h_t, for $t = 1, \ldots, T$.

8: **lgf – double scalar**

The value of the log-likelihood function at $\hat{\theta}$.

9: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g13fg may return useful information for one or more of the following detected errors or warnings.

ifail $= 1$

On entry, **nreg** < 0,
or **mn** > 1,
or **mn** < 0,
or **iq** < 1,
or **ip** < 0,
or **npar** ≥ 20,
or **npar** has an invalid value,
or **ldcovr** $<$ **npar**,
or **ldx** $<$ **num**,
or **dist** $\neq$ 'N',
or **dist** $\neq$ 'T',
or **maxit** ≤ 0,
or **num** $< \max($**ip**$,$ **iq**$)$.

ifail $= 2$

On entry, **lwork** $<$ $($**nreg** $+ 3) \times$ **num** $+ 3$.

ifail $= 3$

The matrix X is not full rank.

ifail $= 4$

The information matrix is not positive-definite.

ifail $= 5$

The maximum number of iterations has been reached.

ifail $= 6$

The log-likelihood cannot be optimized any further.

ifail $= 7$

No feasible model parameters could be found.

7 Accuracy

Not applicable.

8 Further Comments

None.

9 Example

```
iseed = zeros(4, 1, 'int32');
iseed(1) = int32(111);
igen = int32(0);
rwsav = zeros(9, 1);
rvec = zeros(40, 1);
num = int32(1500);
param = [0.1; -0.3; 0.1; 0.9];
bx = [1.5;2.5;3];
dist = 'N';
ip = int32(1);
iq = int32(1);
nreg = int32(2);
mn = int32(1);
df = 5;
mean = 3;
theta = [0.05;
     -0.15;
     0.05;
     0.45;
     1.5;
     0.75;
     1.25];
hp = 0;
copts = true;
maxit = int32(50);
tol = 1e-05;
x = zeros(num, 3);
for i = 1:num
  fac1 = double(i)*0.01;
  x(i,1) = 0.01 + 0.7*sin(fac1);
  x(i,2) = 0.5 + fac1*0.1;
  x(i,3) = 1.0;
end
[igen, iseed] = g05kb(igen, iseed);

fcall = true;
[ht, yt, rvec, iseed, rwsav, ifail] = ...
     g05hn(dist, int32(800), ip, iq, param, df, fcall, rvec, igen, iseed,
rwsav);
fcall = false;
[ht, yt, rvec, iseed, rwsav, ifail] = ...
    g05hn(dist, num, ip, iq, param, df, fcall, rvec, igen, iseed, rwsav);

for i = 1:num
  xterm = 0;
  for k = 1:nreg
    xterm = xterm + x(i,k)*bx(k);
  end
  if (mn == 1)
    yt(i) = mean + xterm + yt(i);
  else
    yt(i) = xterm + yt(i);
  end
end

[thetaOut, se, sc, covar, hpOut, et, ht, lgf, ifail] = ...
```

```
    g13fg(dist, yt, x, ip, iq, nreg, mn, theta, hp, copts, maxit, tol)

thetaOut =
    0.1153
   -0.3097
    0.1209
    0.8936
    2.8629
    1.4524
    2.5812
se =
    0.0202
    0.0268
    0.0444
    0.0143
    0.0948
    0.0588
    0.0622
sc =
    0.7640
   -0.2156
   -0.0441
    0.9403
    0.1838
    0.3071
    0.1796
covar =
    0.0004   -0.0003    0.0003   -0.0003   -0.0005    0.0000   -0.0000
   -0.0003    0.0007   -0.0003    0.0002   -0.0002   -0.0002    0.0002
    0.0003   -0.0003    0.0020   -0.0003   -0.0006   -0.0004    0.0004
   -0.0003    0.0002   -0.0003    0.0002    0.0001   -0.0001    0.0000
   -0.0005   -0.0002   -0.0006    0.0001    0.0090    0.0028   -0.0053
    0.0000   -0.0002   -0.0004   -0.0001    0.0028    0.0035   -0.0018
   -0.0000    0.0002    0.0004    0.0000   -0.0053   -0.0018    0.0039
hpOut =
    3.1436
et =
     array elided
ht =
     array elided
lgf =
   1.5708e+03
ifail =
           0
```